Generalized Coordinates

Dr. Sunil Kumar Yadav*

Department of Physics, Maharaja College, Arrah, Bihar 802301, India.

(Dated: July 27, 2020)

"What we observe as material bodies and forces are nothing but shapes and vibrations in the structure of space."

— Erwin Schrödinger (1887-1961)

In order to specify the position of a particle (in 3D) we need three coordinates (See Fig. (1). Depending on the choices of the coordinate system the coordinates (or variables) will be different, e.g., these will be x, y, and z for Cartesian coordinates; ρ, θ, and z for right circular cylindrical coordinates; r, θ, and ϕ for spherical coordinate or one can choose any other coordinate (like elliptical coordinate) depending on the symmetry of the system. If we have some constraint on the motion of the particle, then the number of coordinates will be less than the earlier case when no constraint are present. Generally, constraints for a given system are defined in terms of equations. For, example if the particle is constrained to move on the plane surface, only two coordinates will be sufficient to describe the motion of the particle.

Similarly, if we consider a N particle system, we need $3N$ coordinates to describe the system. If there is m constraint (equations), then the minimum number of coordinates, n (say), to describe the motion of the system or configuration of the system is given by

$$n = 3N - m. \tag{1}$$

It is to be noted that n is the number of *degrees of freedom* of the system. This is the *minimum number of independent coordinates* needed to completely describe the system. Any set of quantities which completely describes the state or configuration of the system is called

*Electronic address: sunil.phy30@gmail.com

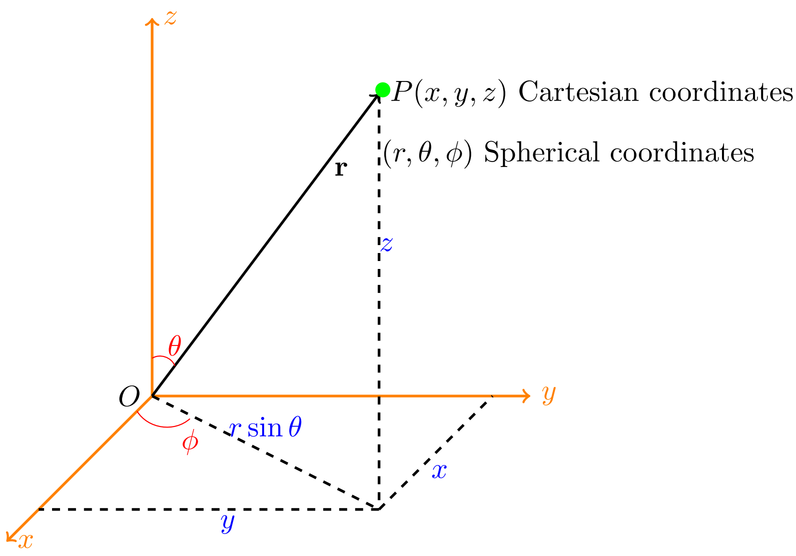

FIG. 1: Cartesian and Spherical Coordinates

generalized coordinates. The generalized coordinates are not restricted by any constraint, they are independent. Generalized coordinates conventionally written as

$$q_1, q_2, q_3, ...q_n,$$

or q_k where $k = 1, 2, 3, ..., n$.

There are two types of constraints: holonomic and nonholonomic.

(a) *Holonomic constraint.* They can be expressed in terms of the equations of the form

$$f(\mathbf{x}_1, \mathbf{x}_2, ..., t) = 0. \tag{2}$$

For example, a particle constrained to move on the circumference of the circle can be completely described by only one coordinate $q = \theta$. The constraint in the form of equation is given by

$$|\mathbf{r}| - a = 0, \tag{3}$$

where $\mathbf{r}$ is the position vector of the particle at angular coordinate θ relative to the center of the circle and a is its radius.

The nonholonomic constraints cannot be expressed in terms of Eq.(2). A sphere restricted to roll on a perfectly rough plane is an example of a nonholonomic constraint. In nonholonomic system coordinates cannot vary independently. Therefore, degrees of freedom is less than the minimum number of coordinates to describe the state of the system. In our discussion, we mainly focus on the holonomic systems.

---

[1] D.S. Mathur, *Mechanics*, S. Chand & Company Ltd., New Delhi, (2007) .

[2] H. Goldstein, C. Poole, and J. Safko,*Classical Mechanics*, Addison-Wesley, (2000).